Use of a grating interferometer for the demodulation of "white-light" interferometric sensors

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ABSTRACT

In order to demodulate optical fiber interferometric sensors in the coherence multiplexing scheme it is worthwhile using static interferomers. Measurement of a physical parameter is obtained by matching the optical path difference (OPD) of each interferometric sensor with a receiver. An attractive solution consists in using grating interferometer as a demodulator. Indeed, it permits one to record correlation peaks (each one corresponding to a sensor) on a photodiode array, thus making demultiplexing easier. On the other hand, it is possible to record only the peak envelope if the grating interferometer is adjusted in a well defined configuration. In this paper we concern ourselves with the theoretical determination of the conditions needed to simplify calculation and interpretation of the experimental results.

1. INTRODUCTION

Optical fibre based interferometric sensors using spectral modulation encoding techniques need efficient demodulation to make demultiplexing easy and the sensor optical path difference (OPD) measurement simple, with few pixels. A solution is provided by a polarimetric interferometer [1] in which birefringent plates are switched to match the optical path difference of a given sensor. Another solution is to use an interferometer whose OPD is not a constant in the exit pupil plane of the interferometer.[2] This solution is more interesting for real time sensing as it permits 'simultaneous' recording of the response of all the sensors on an photodetector array. We will show that the *Spectromètre Interférentiel à Sélection par l'Amplitude de Modulation* (S.I.S.A.M.) [3] or grating - interferometer eliminates most of the drawbacks of other interferometric demodulators. The next section will be concerned by the description of the device and the presentation of the theoretical background with the determination of the conditions which simplify the calculation and thus the interpretation of the experimentally observed signals [4].

2. THE DEVICE

First studied and developed by Connes[3] the SISAM has been used for spectroscopy purposes. It consists of a Michelson interferometer with identical gratings placed in the two arms instead of plane mirrors (Fig. 1.). These gratings are tilted so that they may be near the first order (-1) Littrow mount for the incident collimated beam. An incoherent broadband source with spectral intensity distribution $F(\sigma)$, central wave number σ_0 and full width at half maximum (FWHM) $\delta\sigma$ is coupled to network of sensors. The light emerging from the sensors is linked to the demodulator via an optical fibre. Each elementary wave -assumed to be scalar- forming the incident beam is characterised by its wave number σ and its complex amplitude $a(\sigma)$. These waves impinge on gratings under an incidence angle θ_1. They are thus diffracted by the metallic gratings in the different reflected orders. All the reflected orders except the -1 order are eliminated by spatial filtering. The grating's formula gives us the angle θ of the -1 order reflected wave with the wave number σ :

$$\sigma \sin \theta = \sigma \sin \theta_1 - 1/d \qquad (1)$$

where d is the groove spacing of the gratings. We define σ_1 as the littrow wave number for the incidence angle θ_1 i.e. $\theta = -\theta_1$, it is given by:

$$2 \sigma_1 \sin \theta_1 = 1/d \qquad (2)$$

The previous expression leads to the following rewriting of eq. (1) :

$$\sigma \sin \theta = (\sigma - 2 \sigma_1) \sin \theta_1 \qquad (3)$$

The wave reflected by the grating G_1 and then transmitted by the splitting cube can be written in the reference frame of the beam splitter which assumed to be perfect. If we denote the associated coordinates by X and Y (fig.1.) the reflected wave amplitude $r_1(\sigma,X,Y)$ is given by:

$$r_1(\sigma,X,Y) = 1/2\, a(\sigma)\, R(\sigma) \exp(2\, j\, \pi\, \varphi_1(\sigma,X,Y)) \qquad (4)$$

where $R(\sigma)$ is the reflection coefficient of the -1 order which depends only on the opto-geometric properties of the grating[5]. $\phi_1(\sigma,X,Y)$ is the accumulated phase of the wave propagating from the beam splitter and reflected back to this beam splitter which is deemed achromatic(fig.1.). This phase is given for grating G_1 by [6]:

$$\begin{aligned}\varphi_1(\sigma,X,Y) = &\left[(\sigma-2\sigma_1) \sin\theta_1\cos\theta_1 + \sqrt{\sigma^2 - (\sigma-2\sigma_1)^2\sin^2\theta_1}\ \sin\theta_1\right] X \\ &+ \left[- (\sigma-2\sigma_1) \sin^2\theta_1 + \sqrt{\sigma^2 - (\sigma-2\sigma_1)^2\sin^2\theta_1}\ \cos\theta_1\right] (Y + 2\, \Delta_1)\end{aligned} \qquad (5)$$

with Δ_1 being the length of the first arm.
After reflection on the grating G_2 and on the beam splitter, similar expressions are obtained in the second arm for reflected wave amplitude $r_2(\sigma,X,Y)$:

$$r_2(\sigma,X,Y) = 1/2\, a(\sigma)\, R(\sigma) \exp(2\, j\, \pi\, \varphi_2(\sigma,X,Y)) \qquad (6)$$

with the accumulated phase given by:

$$\begin{aligned}\varphi_2(\sigma,X,Y) = &\left[-(\sigma-2\sigma_1) \sin\theta_1\cos\theta_1 + \sqrt{\sigma^2 - (\sigma-2\sigma_1)^2\sin^2\theta_1}\ \sin\theta_1\right] X \\ &+ \left[- (\sigma-2\sigma_1) \sin^2\theta_1 + \sqrt{\sigma^2 - (\sigma-2\sigma_1)^2\sin^2\theta_1}\ \cos\theta_1\right] (Y + 2\, \Delta_2)\end{aligned} \qquad (7)$$

Δ_2 is the length of the second arm.

On the first assumption, the spectral dependance of the detector response is neglected, so that the total intensity received by a detector placed at point (X,Y) is simply given by the sum of the intensities of all the waves σ and the detected signal S(X,Y) is then proportional to:

$$S(X,Y) = \int_{-\infty}^{+\infty} |a(\sigma)\ R(\sigma)|^2\ |\exp[2j\pi\ \varphi_1(\sigma,X,Y)]\ +\ \exp[2j\pi\ \varphi_2(\sigma,X,Y)]|^2\ d\sigma \tag{8}$$

By looking at eqs.(5) and (7) we show that the signal S does not depend on the Y coordinate : thus it will be denoted by S(X). At this point, we have made the following assumptions : the gratings are identical and the incidence angle is the same on both of them. The beam splitter is considered perfect, its transmission coefficient is equal to its reflection coefficient and these coefficients are independent of the incidence angle and of the wave number σ. Further simplifications come from assumptions about the characteristics of incident light. The last expression (eq. 8) can be simplified if the spectral width $\delta\sigma$ is very small compared to the Littrow wave number σ_1. This implies that this spectral width is also very small if compared to the central wave number σ_0 of the incident light. The phases (eqs 5 and 7) can thus be rewritten to the first order in $(\sigma-\sigma_1)$ to give :

$$\varphi_1(\sigma, X,Y) = 2\ (\sigma-\sigma_1)\tan\theta_1\ X\ +\ \sigma\ (Y + 2\Delta_1) \tag{5'}$$

and

$$\varphi_2(\sigma, X,Y) = -2\ (\sigma-\sigma_1)\tan\theta_1\ X\ +\ \sigma\ (Y + 2\Delta_2) \tag{6'}$$

These relations are those used in the Connes's original papers [3,4]. Introducing these last simplified expressions in eq.(8) and according to small spectral width of the incident light which led us to neglect the σ dependance of grating reflection coefficient $R(\sigma)$, we get:

$$S(X) = \int_{-\infty}^{+\infty} |a(\sigma)|^2\ \left[1 + \cos(\ 8\pi(\sigma-\sigma_1)\tan\theta_1\ X + 4\pi\sigma\Delta\ )\right]\ d\sigma \tag{9}$$

where Δ will be the optical path difference $(\Delta_1-\Delta_2)$ of the demodulator if the dispersion is ignored. The incident intensity factor $|a(\sigma)|^2$ is linked to the source spectral distribution $F(\sigma)$ through the interferometric sensor intensity transmission coefficient [1,4] :

$$|a(\sigma)|^2 = F(\sigma)\ (1 + \cos\ 2\pi\sigma\Delta_s\ ) \tag{10}$$

The interferometric sensor is assumed to beperfect : the visibility is independent of σ and so is the sensor optical path difference Δ_s. To give practical results, the intensity distribution $F(\sigma)$ is chosen as gaussian function of σ with central wave number σ_0 and full width of $\delta\sigma$. This function is a good description of Light Emitting Diode (LED) intensity.

$$F(\sigma) = F_0 \exp\left\{-\left[(\sigma - \sigma_0) / \delta\sigma\right]^2\right\} \tag{11}$$

Let us define the function G of two variables X and X_0 by:

$$G(X,X_0) = \exp\left\{-[\pi\, \delta\sigma\, (X_0 + 2 \tan \theta_1 X)]^2\right\} \cos[4\pi\sigma_0 X_0 - 8\pi \tan \theta_1\, (\sigma_0\text{-}\sigma_1)\, X] \qquad (12)$$

Taking into account relations (10) and (11) we get from eq.(9) that S(X) is proportionnal to

$$[\,1 + G(X,\Delta) + 0.5\; G(X,\Delta\text{-}\Delta_s) + 0.5\; G(X,\Delta\text{+}\Delta_s)\,] \qquad (13)$$

The signal obtained consists of three peaks whose envelopes are gaussian. The central peak is located at $X = \Delta / (2 \tan\theta_1)$ and its amplitude is twice the amplitude of the two other peaks. These peaks are located symetrically at positions which are a measurement of the wanted Optical Path Difference Δ_s. Looking at expression (12), we show that the peaks are modulated according to the X dependance of the cosine function, but this dependance can be removed. This is the great advantage of using the SISAM as a demodulator. Indeed, if the gratings are adjusted is such a way that the Littrow wave number is as close as possible to the source central wave number ($\sigma_1 = \sigma_0$) the measured signal does not contain any modulation which implies low cost (in pixels) and easier processing to determine the sensor OPD -the physical parameter to be measured-. This property of the SISAM has been recently verified [4].

3. A NUMERICAL EXAMPLE

The example we are dealing with is chosen to model a typical experience. The LED has its central wavelength at 850 nm with a spectral width of 40 nm which gives a coherence length of about 20 µm. The gratings consist of ruled gratings with spacing of 300 grooves per mm. To eliminate the modulation of the response the Littrow angle must be adjusted to 7.3252°. Figure 2 show a typical numerical result where Δ is zero -the SISAM two arms have the same length-. In figure 3 is depicted the result concerning the same device as in figure 2 but the incidence angle is changed to 7.°. Figure 3 shows clearly the modulation term. Figure 4 concerns the same device with the same value for the incidence angle but a different value for Δ. It shows that adjustement of Δ implies change in the OPD range and thus tracking of different sensors.

4. CONCLUSION

We have shown on theoritical basis that the SISAM used as a "white-light" demodulator has many advantages over other schemes and it is well adapted to demodulate sensor network thanks to its demultiplexing simplicity and real time processing ability. The conditions of using the simplified theory are established and can be compared to experimental results to give reliable interpretation.

REFERENCES

[1] M. Lequime, C. Lecot, H. R. Giovannini, and S. J. Huard, Soc. Photo-Opt. Instrum. Eng. **1267**, 288 (1990).

[2] C. Mariller, M. Lequime, Soc. Photo-Opt. Instrum. Eng. **298**, 121 (1987).
[3] P. Connes, Rev. Opt. **38**, 157, (1959); Ibid. **38**, 416, (1959); Ibid **39**, 402 (1960).
[4] H. Giovannini, D. Yeddou, S. Huard, M. Lequime, C. Froehly, Opt. Lett. **18**, N° 23 (1993).
[5] R.Petit in "Electromagnetic theory of gratings", R.PETIT editor, Springer-Verlag, Berlin (1980)
[6] H.Akhouayri, H.Giovannini, to be published.

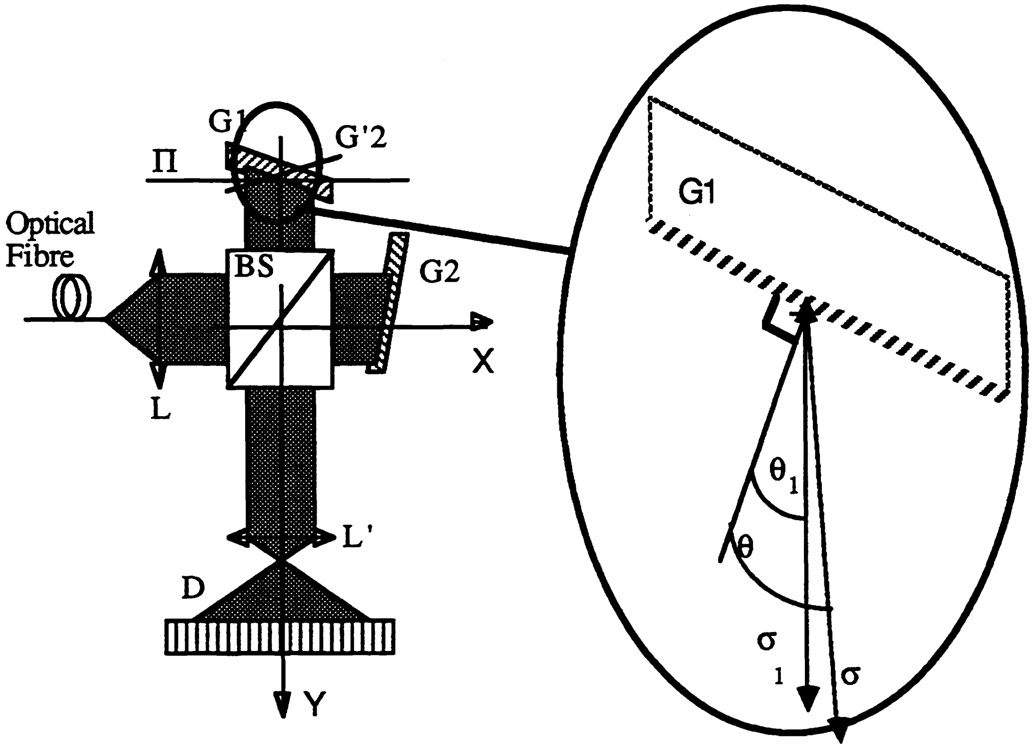

Figure 1. The grating interferometer or SISAM.

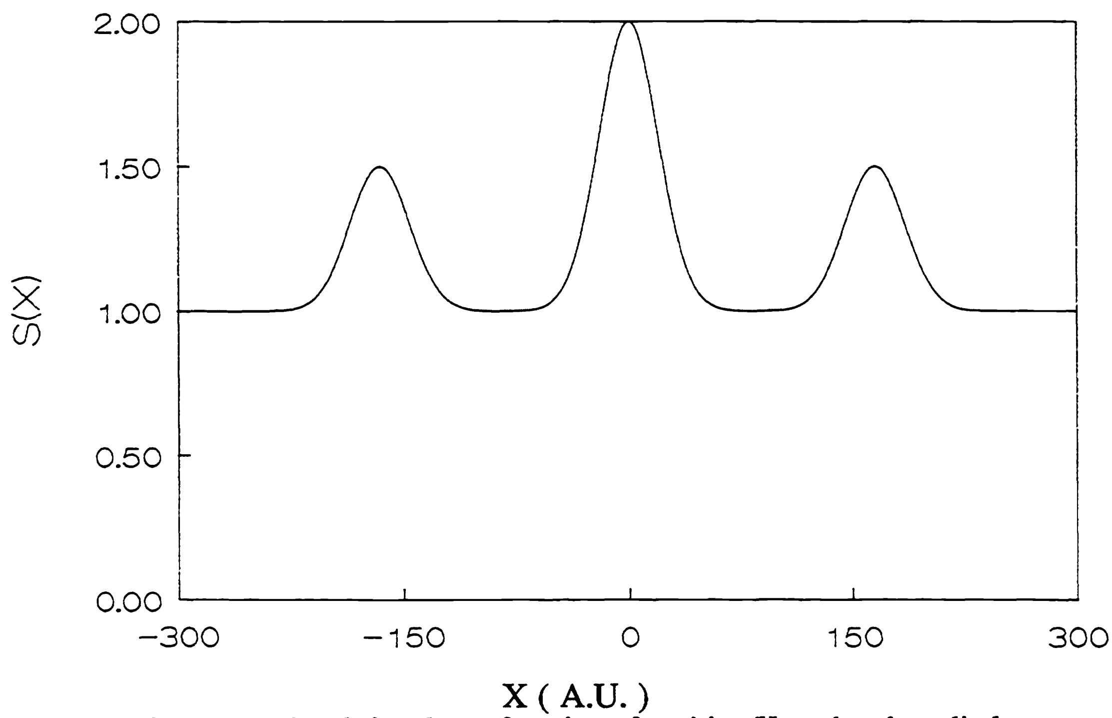

Figure 2. Demodulated signal as a function of position X on the photodiode array. The sisam is adjusted to suppress the modulation ($\sigma_1 = \sigma_0$) and Δ=0.

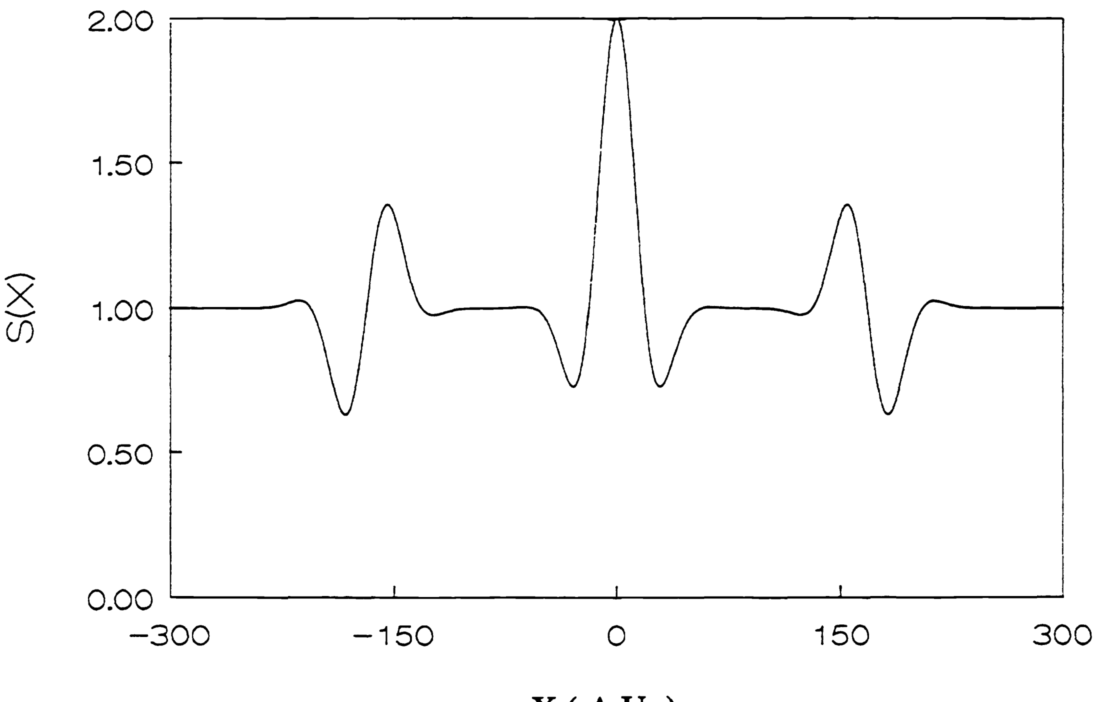

Figure 3. Demodulated signal as a function of position X on the photodiode array. (σ_1 not equal to σ_0) and Δ=0.

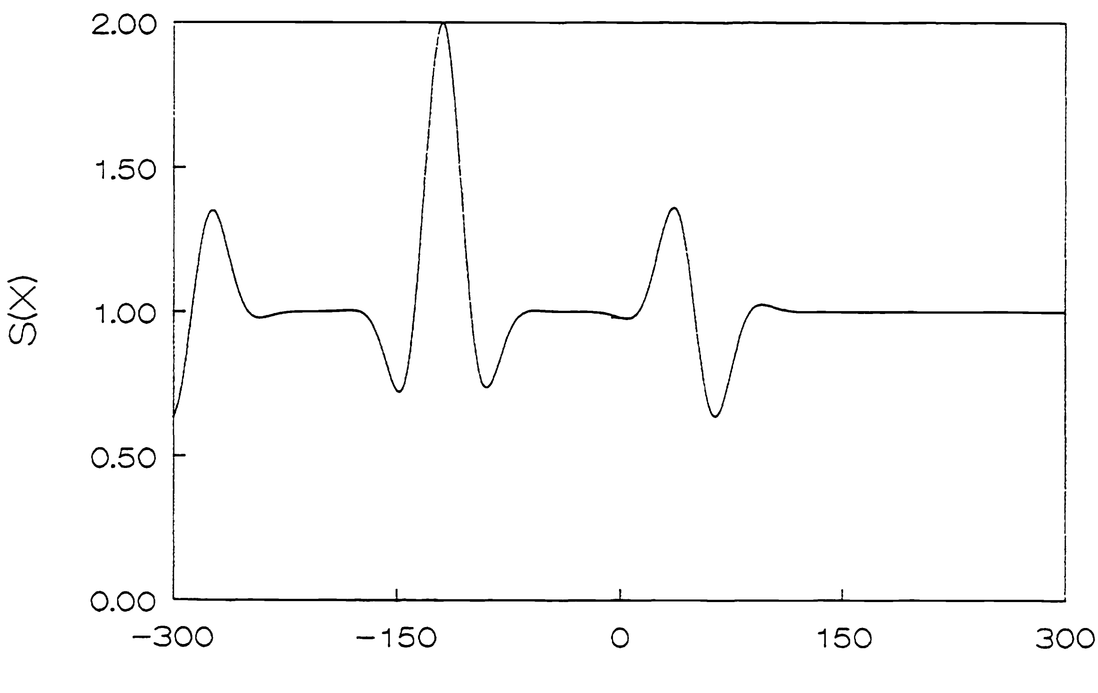

Figure 4. Demodulated signal as a function of position X on the photodiode array. (σ_1 not equal to σ_0) and Δ non zero.